Geometric Phase Locked in the Nucleus

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Abstract

In this paper it is proposed, that nuclear interaction and charge quantization could be modelled by a low-dimensional non-linear iterative system, where instability induced by chaotic dynamics and bifurcations increase with charge or feedback strength. The system could be balanced by a generalized geometric phase part of the electromagnetic coupling strength and Sommerfeld fine structure constant carrying a screening Berry phase component.

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Linear quantum mechanics is probably not the proper method to approach nuclear behavior. Due to a lack of knowledge, nuclear theories including QCD tend to define and postulated 'second level' constructions and abstractions that cannot directly be observed. This can lead to an inflation of redundant parameters and dimensions. It is the success of math to find shortcuts or 'first level' topological and geometrical constructions that can be directly verified and involve fewer dimensions, Berry's phase is a good example. Generally, phase factors or phases representing the 'holonomy' provide for important boundary conditions while reducing the degree of redundancy in variables, especially the phase shifts generated by Berry's connection [1]. This reduction can lead to a dramatic decrease in phase space dimensionality and is one of the reasons why phases and gauge theories are not unimportant in quantum mechanics, despite of the central role of amplitude densities. The non-adiabatic generalization of [2] defines a geometric phase factor for any cyclic evolution of a quantum system, for an introduction see i.e. [3]. It is likely, that spin-orbit coupling in the nuclear and atomic dimensions is a generator for geometric phase shifts since

- it involves fast/slow vector couplings,
- on round trips on a curved surface,
- with exact frequency and phase relationships.

If a geometric phase component generates spin precession, it can couple back to the round trip frequency or path length generating an non-linear feedback loop, especially in the low-dimensional range. In the previous paper [4] such a quantum feedback mechanism has been defined including generalized fine structure constants. Balanced by Berry's generalized geometric phase, nuclear interaction and charge quantization will be modelled by a low-dimensional non-linear iterative system. Instability is induced by chaotic dynamics and bifurcations increasing with charge or feedback strength. Controlled by external fields, the geometric phase has a passive role, but in a phase-locked feedback loop the geometric phase has a double role: it is generated on the closed path and controls the closed path length and dynamic phase. Consequently, it is very interesting to consider round trips of low-dimensional vector signals additionally constrained by the precession dynamics induced by an emerging geometric phase.

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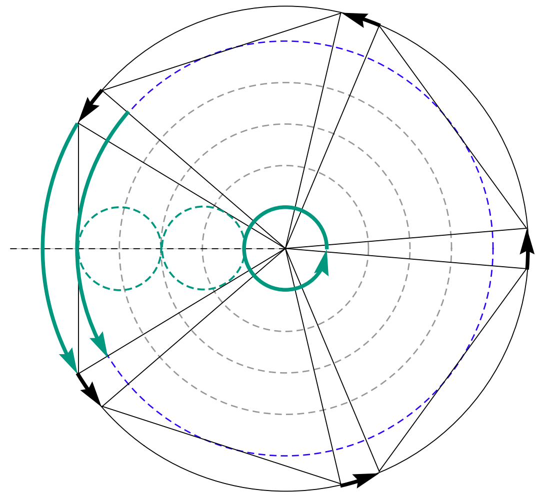

Figure 1: The fine structure M-resonance: all green arrow line segments have the length of the precession cone vertex angle $2\pi\cos(\theta)/M = 2\theta = M_g \triangle\varphi_d(T)$, drawn for $M = 5$ and $M_g = 1$. The two outer circles show the additional contribution of Berry's phase, see the short black arrows with length $\triangle\varphi_g(T)$.

Generalized Berry phase

The non-adiabatic generalization of [2] defines a geometric phase factor for any cyclic evolution of a quantum system (experimentally verified e.g. in [5]). Consider a T-periodic cyclic vector $|\,\psi(\tau)\,\rangle$ that evolves on a closed path $\mathcal{C}$ according to

$$|\,\psi(T)\,\rangle = e^{i\varphi(T)}\,|\,\psi(0)\,\rangle, \tag{1}$$

where the total phase $\varphi(T)$ acquired by the cyclic vector can naturally be decomposed into a geometric $\varphi_g(T)$ and dynamical phase $\varphi_d(T)$

$$\varphi(T) = \varphi_g(T) + \varphi_d(T). \tag{2}$$

The dynamical phase for one loop $t \in [0;T]$ is with the Schrödinger equation given by

$$\varphi_d(T) = -\frac{1}{\hbar}\int_0^T \langle\,\psi(\tau)\,|H(\tau)|\,\psi(\tau)\,\rangle d\tau. \tag{3}$$

The Berry phase or geometric phase depends not on the explicit time dependence of the trajectory and is for one loop given by

$$\varphi_g(T) = i\oint_{\mathcal{C}}\langle\,\psi(\tau)\,|d|\,\psi(\tau)\,\rangle. \tag{4}$$

The 'parallel transported' spin vector will come back after every loop with a directional change $\varphi_g(T)$ equal to the curvature enclosed by the path $\mathcal{C}$. On the unit sphere the curvature increment is proportional to the area increment that can be a spherical triangle with area given by

$$d\Omega := [1-\cos\theta(\tau)]d\varphi(\tau), \tag{5}$$

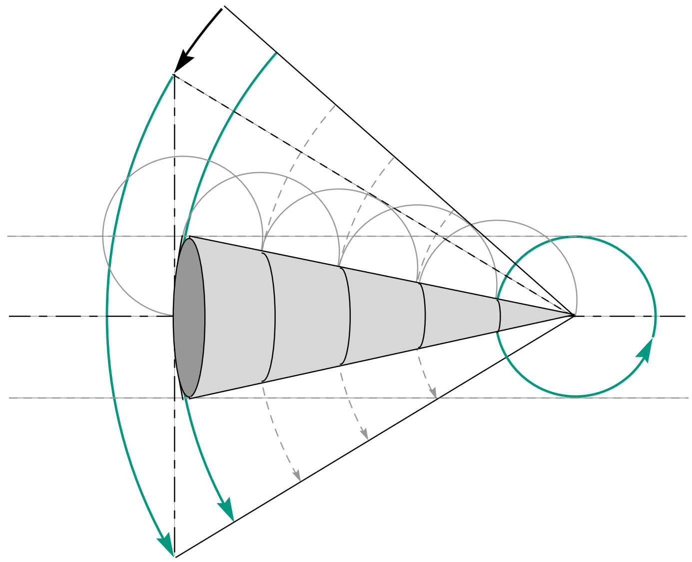

Figure 2: The rolling cone M-resonance: the base of the cone has radius θ/π (small circles), the side length is $M\theta/\pi = \cos(\theta)$. The two green arrow line segments have the length of the precession cone vertex angle and circumference of the cone base $2\pi\cos(\theta)/M = 2\theta = M_g \triangle\varphi_d(T)$, drawn for $M = 5$ and $M_g = 1$. The Berry phase is given by the short black arrow with length $\triangle\varphi_g(T)$.

the total area enclosed by the closed orbit (loop) is equal to

$$\Omega = \oint_{\mathcal{C}} d\Omega := \int_0^T d\tau[1 - \cos\theta(\tau)]\dot{\varphi}(\tau). \tag{6}$$

The Berry phase $\varphi_g(T) = J\Omega$ and the total phase are proportional to spin J. In the standard case of precession on the sphere

$$\varphi_g(T) = 2\pi J(1 - \cos\theta), \quad \varphi(T) = 2\pi J, \tag{7}$$

where θ is the vertex cone semiangle, $\varphi(T) = 2\pi J$, $\varphi_d(T) = 2\pi J \cos\theta$. With n parameters $\lambda_i(t)$, $i = 1, 2, ..., n$ that span a closed curve $\mathcal{C}$ in the T-periodic parameter space $\lambda_i(0) = \lambda_i(T)$, the Berry phase may be represented in terms of the 'gauge potential'

$$A_i = i \langle \psi | \frac{\partial}{\partial\lambda_i} | \psi \rangle, \tag{8}$$

$$\varphi_g(T) = \oint_{\mathcal{C}} A = \int_{\mathcal{S}_{\mathcal{C}}} F, \quad F = dA, \tag{9}$$

where A can be regarded as a winding number density, $\mathcal{S}_{\mathcal{C}}$ is an arbitrary surface in the parameter space bounded by the contour $\mathcal{C}$. For more details regarding monopoles and Wilson loops on the lattice in non-Abelian gauge theories, see e.g. [6]. Note if the Berry phase contains a string-like singularity (Dirac string) somewhere on the surface of $\mathcal{S}$, the position of the singularity may be arbitrary shifted by U(1) gauge transformations. In the first step we will introduce a simple feedback relation of the geometric phase to the dynamical phase. The mostly important question is, what balances both parts of the total phase, what is the phase boundary condition? It is likely that iterative round trips of vector signals include a geometric phase component. If so, this component will couple back to the round trip frequency or path length generating an non-linear feedback loop (i.e. induced by precession). Berry's phase appears if the fast dynamics (spin sub-loops) affects the slow dynamics (orbital loop) and vice versa. The coupling between both dynamics can be quantified in terms of an orbital frequency and Compton frequency of a spinning particle, where the ratio in the ground state is usually defined as the fine structure coupling constant. A 'rolling cone' representing a vector state or signal is probably the simplest model of spin-orbit coupling. Rotated once, the cone will change its orbital orientation by a special angle $2\pi/M$, rotated M-times in the quantum case, the cone will return to the initial position with integral M (providing for single-valuedness). Visualizing Thomas precession and aberration (angle θ obtained by infinitesimal Lorentz boosts) [7] already pointed out, that the geometric phase can be found in classical mechanics with a gyroscope or point-like compass as a solid-body turn during conical movement [8]. The correspondent cone geometry is shown in fig.2.

Fine structure iteration

The question is, what balances both parts of the total phase? The frequency ratio in a phase-locked situation follows the requirement of single-valuedness and provides for an integral number of M spinning periods on one round trip path (slightly modified by precession). Starting with this model, M can divide the total phase range (slow orbit) into M sub-loop intervals (fast spin) $\triangle\varphi(T) = \triangle\varphi_d(T) + \triangle\varphi_d(T)$ where

$$\varphi_d(T) = M\triangle\varphi_d(T), \quad \varphi_g(T) = M\triangle\varphi_g(T), \quad M = \pm 1, \pm 2, ... \tag{10}$$

The effect of precession can be a phase modulation of the orbital path length that could couple to the number of sub-loops by modulating the 'rolling cone path'. Intuitively it should be clear how precession could directly "modulate" the path length and phase on a round trip. In any case it is a non-linear feedback situation where the precession angle will increase with increasing curvature enclosed by the path. The rolling cone resonance, see figs.1 and 2, provides for the feedback relation

$$2\theta = M_g\triangle\varphi_d(T), \tag{11}$$

an orbital resonance condition regarding the dynamical phase and the precession phase. If the cone rotates once, both, the radial and orbital waves will couple back in-phase because of the integral wavenumber on the radial and orbital paths. Now it is possible to find with eq.(7), eq.(10), and eq.(11) the optimum θ for a given M and J, where

$$M\theta = JM_g\pi\cos\theta. \tag{12}$$

As a test for $J = M_g = 1$ and $M > 0$, eq.(12) can be solved by iteration

$$\theta_{i+1} = \frac{\pi\cos\theta_i}{M}. \tag{13}$$

After a few steps the algorithm converges (no problem for $JM_g \ll M$).

The criterion for the convergence of the nonlinear affine iterative system or the asymptotic stability of the fixed point will be visualized in the next sections showing bifurcations and unstable regimes.

Frequencies

Let ω_M be the orbital evolution of the dynamical phase on a circular geometry. The spin-rotation coupling will act on the spinning particle with mass-energy $E = \hbar\omega$ via precession

Table 1:
Convergent fine structure (re)generation constants α for $Z_e = 1$ and variable $M > 2$. The third row shows $N = |\triangle\varphi_d(T)/\triangle\varphi_g(T)|$ or $|N_\pm|$ (bottom), known as winding number on helical paths.

M	J/α	N
3	4.13669	2.63924
4	4.96178	4.15896
5	5.82662	6.04873
6	6.72097	8.32214
7	7.6371	10.98727
137	**137.03600941164**	3804.560912
137	**137.03600998817**	3804.5[1]
137	**137.03600052556**	3805.5[1]
137	**137.03599106791**	3806.5[1]
137	137.03598161523	3807.5[1]

[1] The next hypocycloidal or epicycloidal resonances for $M = 137$ instead of free running ratio N between dynamical and geometric phase. The values are discussed and compared to measurement [11] in [4].

energy $E_p = \bar{\omega}_p$. The relative dynamical coupling strengths can be defined by the ratio

$$\alpha(M) = \frac{J\triangle\varphi_d(T)}{\varphi(T)} = \frac{J\omega_M}{\omega}, \tag{14}$$

a generalized fine structure constant, where the coupling is proportional to the evolution of the dynamical part $\triangle\varphi_d(T)/\varphi(T)$ and to spin J. With eq.(10) in eq.(14)

$$\frac{\varphi_g(T)}{2\pi J} = 1 - \frac{M\alpha}{J}, \tag{15}$$

the dynamical part of eq.(15) in eq.(7) provides for

$$M\alpha = J\cos(\theta). \tag{16}$$

Comparing eq.(12) with eq.(16) the precession cone vertex angle 2θ of eq.(7) equals the dynamical phase of the spin-orbit interaction part in eq.(10) with

$$\theta = \pi M_g \alpha. \tag{17}$$

The two possible signs can be combined to $M/M_g > 0$

$$M\alpha = J\cos(\pi M_g \alpha), \tag{18}$$

and for $M/M_g < 0$

$$M\alpha = J\cos(\pi - \pi M_g \alpha). \tag{19}$$

Results for α with variable M for $M_g = J = 1$ are shown in table 1, visualized inclusive measurements in fig.**??**, and simulated with a Java applet in [12].

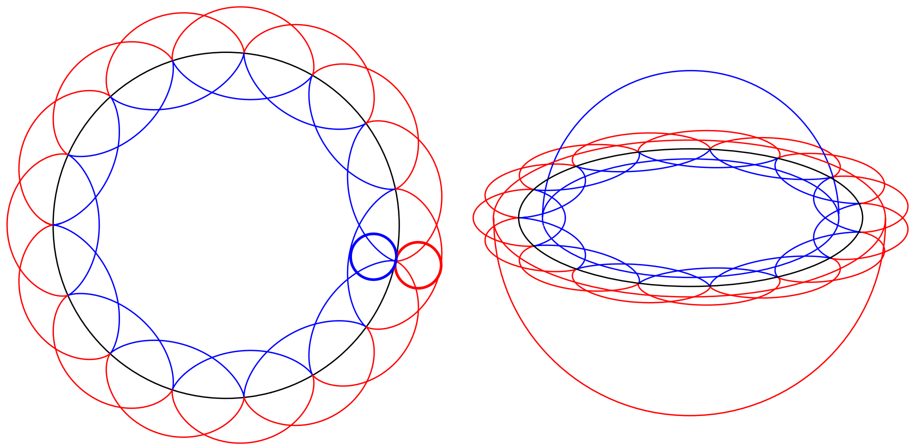

Figure 3: The winding number N-resonance: The spin $\frac{1}{2}$ construction of nonsingular vector potentials for a monopole with charge $M = 5, N_- = N_+ = 7.5$ is given by the connection of epicycloids (outer) and hypocycloids (inner) patches in accordance with Dirac [9], Wu and Yang [10].

It should be noted, that according to eq.(14) and eq.(15) the general expression for spin-rotation coupling observed in the laboratory frame (relativistic correction γ) can be assumed to be

$$\frac{\omega_p}{\gamma\omega} = 1 - M\frac{\omega_M}{\omega}, \tag{20}$$

where the resulting relation

$$E_p = \gamma(E - M\hbar\omega_M), \tag{21}$$

appears exactly in the general form of general relativistic (gravitomagnetic) spin-orbit interaction (Lense-Thirring effect) [13, 14], an integer M covers scalar and vector fields. Usually, the gravitomagnetic effect can be hardly observed because of its tiny magnitude (tests with orbiting gyroscopes are on the way, see gravity probe B news [15]). But the tiny magnitude of the gravitomagnetic field in a classical measurement does not necessarily mean, that the magnitude of the emerging geometric phase and related quantum mass–energy currents in feedback loops must be tiny. Recently, [16] discussed coupling gravitomagnetism-spin and Berry's phase and pointed out, that the geometric phase changes should depend exclusively upon the solid angle of a field, and not on the strength of the field. If gravitomagnetic spin-rotation coupling eq.(21) controls the feedback loop in combination with eq.(18), the mass-energy current could increase to a level that is only limited by damping and (multipole) radiation effects, probably a level characterized by electromagnetism.

Nuclear topology and overload

As shown by Berry, a geometric phase is produced in the field of a magnetic monopole [1]. Magnetic monopoles are topological in nature and are represented geometrically by non-trivial bundles. For electrodynamics, the gauge group is $U(1)$ which has the topology of a circle, on which the homotopy classes of closed curves are labelled by their winding or loop numbers, and where the magnetic charge is quantized taking integral values [9]. In electromagnetism the charges are multiples of a fundamental charge M, so that the wave-function transforms as

$$\psi \rightarrow e^{\pm iM\Lambda}\psi, \tag{22}$$

the unit charge corresponds to the phase sub-interval $[0, 2\pi/M]$. The geometrical phase in quantum mechanics is ultimately related to the construction of an Abelian monopole since this is the only topologically non trivial object which arises when the structure group is U(1)[6]. For appropriately chosen base space magnetic monopoles on $SU(2)/U(1) = S^2$, the resulting fiber bundle has a non-trivial topology with non-zero Chern number. It is the total flux produced by the bundle curvature quantized to values $M/2$ with Hopf invariant M and stereographic projection on S^3. The construction of nonsingular vector potentials for a monopole in accordance with Dirac [9], Wu and Yang [10] is shown in fig.3 for $M = 5$. For the two cases of opposite parity (epicycloidal and hypocycloidal) the Berry phase is evolving with or against the dynamic phase. The gradient of the geometric phase (the gauge potential of the monopole) is given by the radial difference in epicycloids-hypocycloid phase evolution. Dirac [9] showed that the existence of magnetic monopoles can explain the quantization of electric charge and that a monopole must carry a magnetic charge which is an integral multiple of 68.5. According to these monopole properties and the U(1) relation to the Berry phase for $J = \frac{1}{2}$, $M = 137$ is the topological candidate to generate the quantum monopole charges of magnetism and electrostatics. Solving eq.(18) or eq.(19) by iteration provides for the balance of dynamical and geometric phase given by α subject to a given number of sub-loops M, coupling loops M_g, sub-loop spin J. The coupling is polar since a positive and negative M/M_g corresponds to the repulsive and attractive case, respectively, in the negative case the coupling phase interval and precession of eq.(17) is negative with respect to the total phase in eq.(10).

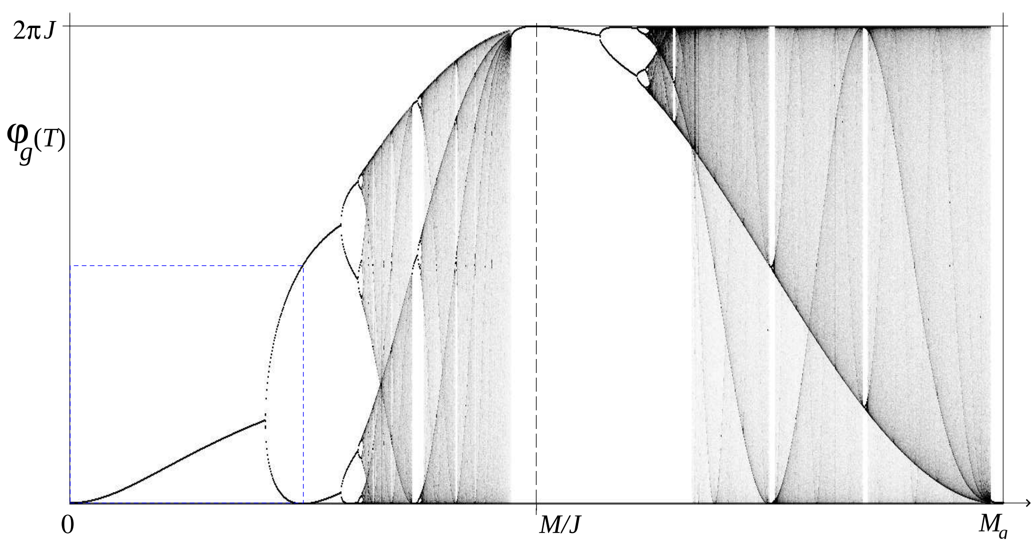

Figure 4: Chaotic occupation density of geometric phase space for a variable coupling number $M_g = Z_e/J$. Stable regions with one fixed $\varphi_g(T)$-value and bifurcations occur periodically. Stable regions for $J = \frac{1}{2}$ and $M = 137$ exist with M_g values 1-114, 260-310, 541-568. Feigenbaum's number ≈ 4.669 characterizes the branching sequence of bifurcations. The dashed blue rectangle shows $M_g \leq 137$ for $J = \frac{1}{2}$.

Increasing $|M_g/M|$ will increase the precession semiangle $|\theta|$ for a given dynamical phase of the sub-loop $\triangle\varphi_d(T)$. With variable M_g, eq.(18) and eq.(19) characterize a complex one-dimensional system that can show chaotic dynamics and quasiperiodicity [17]. It is a cosine map related to the circle and sine map [18] and as an iterative system it shows asymptotic stable and converging regimes but also bifurcations and unstable regimes for special feedback coupling strengths M_g. The geometric part and precession will become more and more dominant with increasing M_g, blocking or occupying phase space as a charge-dependent screening effect, see fig.4. If the nonlinear system characterizes Coulomb coupling and fine structure, the chaotic dynamics should be found in charged nuclei where M_g reaches the critical value. Since the production cross section was found to be rapidly decreasing with the atomic number near 114 [19], it was concluded that it would be very difficult to reach still heavier elements. In fig.4 the first stable regions with one fixed $\varphi_g(T)$-value end s near $M_g = 114$, the next bifurcations occur periodically. This indicates, that the Coulomb coupling generating chaotic electrodynamics could to some extend be responsible for the instability of the nucleus.

The prototype realization of such a nonlinear system on SU(2)/U(1) = S^2 with Dirac monopole charge $M = 137$ reveals that the coupling strength can be quantized and parametrized as an electromagnetic charge Z. The coupling strength could be quantized and parametrized as an electromagnetic charge $Z = JM_g$ driven by spin, where $J = 1$ for a bosonic vector field. Near $Z = 114$ the geometric iteration becomes unstable and generates the first bifurcation.

The shape of the loop-on-the-loop (i.e. hypocycloids and epicycloids) can be conformally mapped to other paths. Circles and the tangencies are preserved by the fractional-linear transformations of the Riemann sphere. Entering geometer's world, projective geometry enables to map the epicycloids to hypocycloid and vice versa the signal using conformal transformations, i.e. in the more simpler case by inversions at the sphere combined with translations. In the complex plane Möbius transforms are conformal transformation that always map circles on circles (especially rolling circles!), where a circle is either a usual circle or a straight line, i.e., a circle with infinite radius. In the case of central potentials the transition to polar coordinates and vice versa can be very effectively handled by a conformal mappings involving the complex transformations $\log(z)$ and $\exp(z)$.

The winding number N for half spin charges interacting is given in a hydrogen-type ground state by an half integer $N = 3805.5$, changing $N \to N \pm 1$ the fine structure constant is increased with respect to the neutral reference in the hypocycloidal case by $\approx 3805.5^{-2} \approx 6.9 \cdot 10^{-8}$ and in the epicycloidal case reduced by approximately the same part [4]. Regarding the α-powers produced with N^2

$$\alpha^2 \approx \frac{2}{\pi^2 N}, \tag{23}$$

the Berry contribution with coupling change $\triangle\alpha/\alpha$ proportional to $1/N^2$ has lowest order α^4-terms. It should be interesting to note, that hyperfine, fine structure, and Lamb shift are usually assigned to the same α-power dependency. It is not clear why geometric phase contributions are missing in almost all perturbative QED evaluations of the atomic spectra.

Summary and conclusion

Predicting nuclear behavior requires to understand the nature and source of Coulomb interaction and the probably most prominent fundamental constant. The proposed non-linear loop model subject to single-valuedness provides for interesting results:

- In the quantum regime geometric phase shifts must be included into the (cyclic) boundary conditions, especially iterative round trips of vector signals (spin) on a curved manifold, i.e. SU(N)/U(N-1) = S^2, include a geometric phase component.

- A geometric phase component usually generates spin precession. It is likely that it couples back to the round trip frequency or path length and generates a non-linear feedback loop.

- The feedback strength is a convergence and stability criterion. Chaotic dynamics emerges at a special coupling strength.

Non-linear precession dynamics generated by spin-orbit and spin-spin interaction could be the key to approach nuclear stability, additional details can be found in [4].

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