Magic Angle Precession and the Riemann Zeta Function

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Abstract. The Riemann zeta function and the magic angle precession strange attractor are related by their functional equations. This enables to express the generalized fine structure spin-orbit coupling constants of the chaotic attractor in simple terms of gamma and zeta functions and supports the former conjecture that rotated rotations on curved surfaces have intrinsic computation capabilities. It seems that the monopole coupling emerges between a zeta function and its reflection on a surface with constant curvature and fractional or fractal dimension.

Functional Equations

The reflection functional equation for the Riemann zeta function $\varsigma(s)$ with complex s and gamma function $\Gamma(1+s)$ is given by [1]

$$\cos\left(\tfrac{1}{2}\pi s\right)\frac{\Gamma(1+s)}{(2\pi)^s}\frac{\varsigma(s)}{\varsigma(1-s)}=\tfrac{1}{2}s\,. \tag{1}$$

Magic Angle Precession (MAP) [2], a cosine chaotic map describing rotated rotations on curved surfaces with Dirac quantized monopole charge $M_{\mathbb{N}}\in\mathbb{N}$ and spin-orbit coupling strength α can be written as

$$\cos\left(j\pi\alpha\right)=M_{\mathbb{N}}\alpha\,, \tag{2}$$

where spin times charge = j, precession angle $\theta=j\pi\alpha$, and Berry phase $j\pi(1-M_{\mathbb{N}}\alpha)$. Comparing both functional equations (1) and (2), the generalized fine structure constants can be expressed with $s=2j\alpha$ in simple terms of gamma and zeta functions, where the monopole charge is given by

$$M(s)\to M_{\mathbb{N}}=j(s)\frac{(2\pi)^s}{\Gamma(1+s)}\frac{\varsigma(1-s)}{\varsigma(s)},\;\; s=2j\alpha\,, \tag{3}$$

see fig. 1.

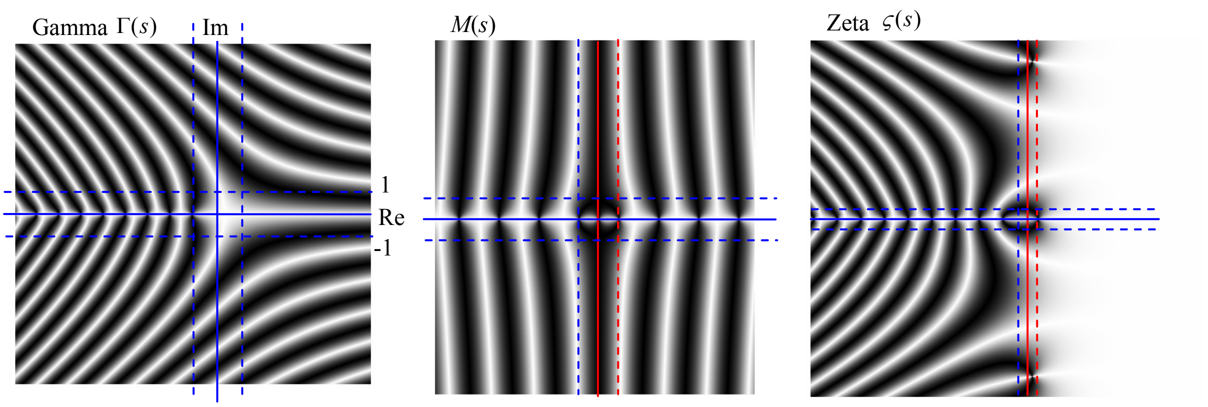

Figure 1: Visualizing complex analytic functions using Lundmark's domain colouring algorithm [4] in the complex plane reveals the functions landscapes. Left the gamma function, mid the MAP coupling function with critical strip $0\le \mathrm{Re}\,s\le 1$ between red lines, right the zeta function with critical strip and first nontrivial zeros pair at the top and bottom.

The range $-2 \le \mathrm{Re}\, s \le 2$ of $M(s)$ is shown in fig.2. Iterating $s_i \to s_{i+1}(s_i)$ for a given quantized $M_{\mathbb{N}}$ and running j the attractor can lead to fixed points, limit cycles, chaos, and bifurcations from

$$M_{\mathbb{N}} s_{i+1} = j(s_i) \frac{(2\pi)^{s_i}}{\Gamma(s_i)} \frac{\varsigma(1-s_i)}{\varsigma(s_i)} . \tag{4}$$

Monopole Gauss Flux

With the solid angle $\Omega(1+s)$ subtended by the full surface of the unit sphere in $1+s$ dimensions we have from (3)

$$M_{\mathbb{N}} = \frac{j(s)}{\left(\frac{1}{2}\right)^s} \frac{\Omega(1+s)}{\pi} \frac{\varsigma(1-s)}{\varsigma(s)} , \qquad \Omega(1+s) = \frac{\pi^{1+s}}{\Gamma(1+s)} . \tag{5}$$

Assuming that the monopole coupling emerges between $\varsigma(s)$ and its reflection $\varsigma(1-s)$ with a Gauss flux exchange through a surface with constant positive curvature in $1+s$ dimensions altering spin and charge, the monopole strength $M_{\mathbb{N}}$ in equation (5) could be decomposed into a product of three terms:

- spin-charge factor $\mathrm{J}(s) = j(s) 2^s$,
- solid angle factor $\Theta(s) = \Omega(1+s)/\pi$, and
- reflection factor $\mathrm{P}(s) = \varsigma(1-s)/\varsigma(s)$. (6)

In terms of definitions (5) and (6) we have

$$M_{\mathbb{N}} = \mathrm{J}(s)\Theta(s)\mathrm{P}(s) . \tag{7}$$

Coupling Properties

The Dirac quantum condition for a real monopole charge number $M_{\mathbb{N}} = 137$ (and Cooper pair version with spin times charge $j = 1$) leads by iteration $s_i \to s_\infty = s$ in (4) to the fixed point with coupling strength $2s^{-1} = \alpha^{-1} = 137.036009...$. It should be noted (just for encouraging further research) that this fixed point of a generalized spin orbit coupling constant α is within measurement accuracy (depending on the underlying theory and given assumptions) the value of the Sommerfeld fine structure constant [2]. The strange attractor in (4) can produce fractals and shows a Feigenbaum-type bifurcation spectrum [2,3]. Spin anomalies related to the magnetic moment are typical side effects of the holonomic effects induced by the Gauss spin flux through a surface with constant positive curvature in $1+s$ dimensions measurable as precession or wobbling.

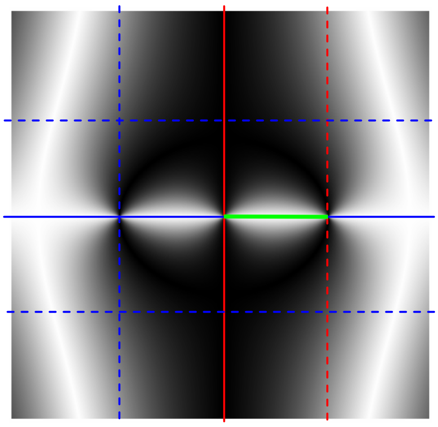

Figure 2: Magnification of fig 1, $M(s)$ critical strip $0 \le \mathrm{Re}\, s \le 1$ between red lines. The real MAP interaction happens on the real line in the critical strip, see the green line.

Outlook

It should be interesting to compare the self-critical fractal power-law bifurcation density [2] and auto-correlation spectrum of MAP (generally rotated and translated loops and rotations on curved surfaces) with some similar properties of the Riemann zeta function rooted in the power-law Mellin transform [1], especially those related to computing [3] and prime counting [1].

Literature

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